

DSC Design For Synchronization And Anti-Synchronization of Arneodo Chaotic System

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Abstract—This paper aims to propose the synchronization and anti-synchronization of identical Arneodo chaotic system (1980) by dynamic surface control design. Dynamic surface control (DSC) is a new approach based on backstepping, which tackles the problem of ‘explosion of terms’ arising due to repeated differentiations in conventional backstepping. It leads to a better approach for synchronization and anti-synchronization of two identical chaotic systems.

Index Terms—chaos, synchronization, anti-synchronization (AS), arneodo system, dynamic surface control (DSC)

1. INTRODUCTION

One of the most common events in various disciplines of science and engineering is chaos. Chaos is a long-term aperiodic behavior in a deterministic system that sensitively depends on the initial conditions. Today, chaos finds many useful applications in many engineering systems such as in chemical reactors, genetic control systems, power converters, lasers, biological systems, and secure communication systems [2],[9]. Chaotic behavior may lead to undesirable effects as well, such as uncontrolled oscillations in a power grid, and may need to be regulated.

Chaos synchronization appears to be difficult to notice in physical systems. Synchronization has been extensively studied by Pecora and Carroll [3], [4]. They introduced a method by which synchronization of two identical chaotic systems with different initial conditions take place. Chaos synchronization can be utilized in the vast area of physics and engineering systems such as in biological systems and chemical reactions [5], [6]. For a system of two identical chaotic system, the master ($\dot{x} = f(x, y)$) and the slave ($\dot{y} = g(x, y)$) where $x(t)$ and $y(t)$ are phase space or state variables. Synchronization implies that $x(t) - y(t) \rightarrow 0$ as $t \rightarrow \infty$.

On the other hand, anti-synchronization of two identical systems ($\dot{x} = f(x, y)$) master system and ($\dot{y} = g(x, y)$) slave system means $x(t) + y(t) \rightarrow 0$ as $t \rightarrow \infty$. This phenomenon has been analyzed both practically and theoretically in various physical systems [7], [8], [11]. A current study of the AS process in non-equilibrium systems advice that AS could be used as a method for particle division in a fusion of interacting particles [10].

In this paper, dynamic surface control (DSC) design is proposed for synchronization and anti-synchronization of identical Arneodo chaotic systems. Conventional backstepping although systematic, suffers from severe problem of explosion of terms which arises due to repeated differentiations of virtual controllers. Thus, the complexity of controller increases severely limiting the use of backstepping technique to higher order systems. In order to tackle this problem, a new approach called dynamical surface control (DSC) was first introduced by Swaroop et al. [12] for non-adaptive systems and further extended by Patric and Hedric [13] for adaptive systems. In DSC, a first order filtering of the synthetic input is carried out at each level of the traditional backstepping design.

The structure of this research paper is as follows. In section 2 there is the description of the arneodo chaotic system. In Section 3, we design a DSC controller for the synchronization of identical Arneodo systems. In Section 4, we design a DSC controller for the anti-synchronization of identical Arneodo systems with the known system parameters respectively.

2. DESCRIPTION OF THE ARNEODO CHAOTIC SYSTEM

Arneodo system is the type of classical 3-D chaotic systems as it entraps many features of chaotic systems. The Arneodo system was proposed by Arneodo, Tresser and Couillet [1] in 1980. As the master system, Arneodo dynamic is described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3(1) \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2\end{aligned}$$

Where the states of the system are denoted by x_1, x_2, x_3 and a, b are positive, real parameters of the system. The chaotic behavior of the Arneodo chaotic system described in Eq.(1) for parameter values $a = 7.5, b = 3.8$ with initial conditions $x_1(0) = 3, x_2(0) = -8$ and $x_3(0) = -2$, is shown in Figure 1. Figure 1 depicts the three dimensional phase portrait of the strange chaotic attractor of Arneodo system.

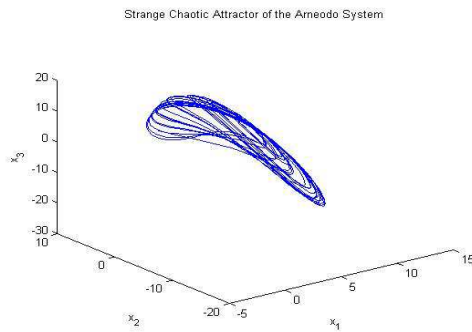


Fig.1. Strange chaotic attractor of Arneodo system

3. DSC DESIGN FOR SYNCHRONIZATION

3.1 Synchronization of Arneodo Chaotic Systems

As system Eq.(1) be the master system and the following system be the slave system of Arneodo dynamic

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3(2) \\ \dot{y}_3 &= ay_1 - by_2 - y_3 - x_1^2 + u \end{aligned}$$

Where the states are denoted by y_1, y_2, y_3 and active control is denoted by u which we have to design. The synchronization error between the master system Eq. (1) and the controlled slave system Eq. (2) is defined as

$$\begin{aligned} e_1(t) &= y_1(t) - x_1(t) \\ e_2(t) &= y_2(t) - x_2(t) \\ e_3(t) &= y_3(t) - x_3(t) \end{aligned} \quad (3)$$

The design problem is to detect $u(t)$ so that the error asymptotically converges to zero, i.e. $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$.

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3(4) \\ \dot{e}_3 &= ae_1 - be_2 - e_3 - (y_1 + x_1)e_1 + u \end{aligned}$$

The objective is to find a control law which will stabilize the system Eq.(4) at the origin. Now, let us define first surface S_1 as

$$S_1 = e_1 - e_{1d} \quad (5)$$

Here, e_{1d} can be taken equal to zero as error e_1 is desired to be zero. The virtual control \bar{e}_2 is chosen so as to drive $S_1 \rightarrow 0$.

$$\bar{e}_2 = -K_1 S_1 \quad (6)$$

Then the first order filter will be,

$$\tau_2 \dot{e}_{2d} + e_{2d} = \bar{e}_2, \quad e_{2d}(0) = \bar{e}_2(0) \quad (7)$$

Again, defining the second surface S_2 as

$$S_2 = e_2 - e_{2d} \quad (8)$$

Choosing \bar{e}_3 to drive $S_2 \rightarrow 0$

$$\bar{e}_3 = -K_2 S_2 + \dot{e}_{2d} \quad (9)$$

And e_{3d} can be obtained by filtering \bar{e}_3 .

$$\tau_3 \dot{e}_{3d} + e_{3d} = \bar{e}_3, \quad e_{3d}(0) = \bar{e}_3(0) \quad (10)$$

Next defining the third surface S_3 as

$$S_3 = e_3 - e_{3d} \quad (11)$$

Finally, as already explained in previous section, to drive $S_3 \rightarrow 0$, the controller for the above system can be chosen as

$$u = \dot{e}_{3d} - (ae_1 - be_2 - e_3 - (y_1 + x_1)e_1) - K_3 S_3 \quad (12)$$

For the verification of the efficacy of the proposed approach of synchronization, the simulation result is shown.

3.2 Simulation Results

The system described by Eq. (1), in section 2 is considered here for simulation purpose. The parameters a and b are taken as earlier which were $a = 7.5$ and $b = 3.8$. The initial conditions for master and slave system are taken as $x_1(0) = 3, x_2(0) = 8, x_3(0) = -2$ and $y_1(0) = 10, y_2(0) = -4, y_3(0) = 7$, respectively. The values of time constants τ_2 and τ_3 are taken as: 0.4 and 0.2 respectively, and the surface gains are selected as $K_1 = 1, K_2 = 2$ and $K_3 = 3$. Synchronization behaviour of master and slave Arneodo chaotic system for the control strategy defined in Eq.(12) is shown in Figure 2. Synchronization of states $x_1 y_1, x_2 y_2$ and $x_3 y_3$ can be observed in Figure 2(a). The convergence of state errors to zero in finite time is evident from Figure 2(b), and thus, it can be concluded that the master and slave system get synchronized in finite time.

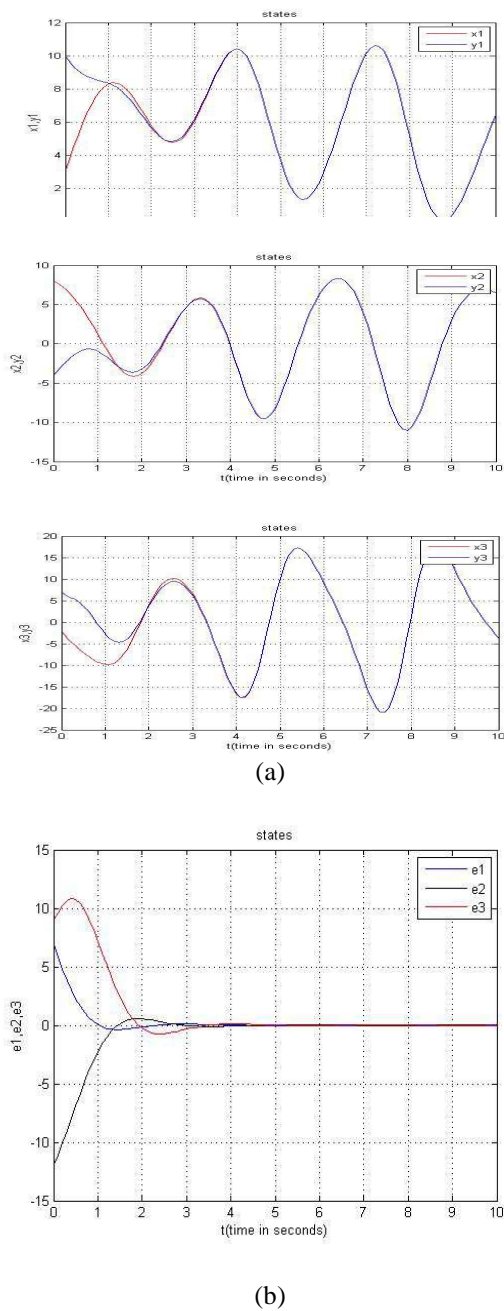


Fig.2. Synchronization of Arneodo system: (a) Synchronization of states and (b) Convergence of errors

4. DSC DESIGN FOR ANTI-SYNCHRONIZATION

4.1 Anti-Synchronization of Arneodo Chaotic Systems

As system of Eq. (1) be the master system and the following system be the slave system of Arneodo dynamic

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = y_3(13)$$

$$\dot{y}_3 = ay_1 - bx_2 - x_3 - x_1^2 + u$$

Where the states are denoted by y_1, y_2, y_3 and active control is denoted by u which we have to design. The anti-synchronization error between the master system Eq.(1) and the controlled slave system Eq.(2) is defined as

$$e_1(t) = y_1(t) + x_1(t)$$

$$e_2(t) = y_2(t) + x_2(t)(14)$$

$$e_3(t) = y_3(t) + x_3(t)$$

The design problem is to detect $u(t)$ so that the error asymptotically converges to zero, i.e. $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$.

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = e_3$$

$$(15) \dot{e}_3 = ae_1 - be_2 - e_3 - y_1^2 - x_1^2 + u$$

The rest of the steps taken from Eq.(5) to Eq.(11) also follow exactly.

Finally, as already explained in previous section, to drive $S_3 \rightarrow 0$, the controller for the above system can be chosen as

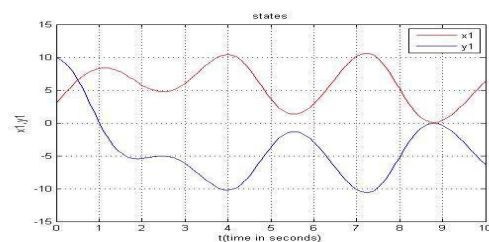
$$u = \dot{e}_{3d} - (ae_1 - be_2 - e_3 - y_1^2 - x_1^2) - K_3 S_3$$

$$(16)$$

For the verification of the efficacy of the proposed approach of synchronization, the simulation result is shown.

4.2 Simulation Results

Parameters and initial conditions are same as in the previous section. The values of time constants τ_2 and τ_3 are taken as: 0.4 and 0.2 respectively, and the surface gains are selected as $K_1 = 1, K_2 = 2$ and $K_3 = 3$. Anti-synchronization behaviour of master and slave Arneodo chaotic system for the control strategy defined in Eq.(16) is shown in Figure 3. Anti-synchronization of states x_1, y_1, x_2, y_2 and x_3, y_3 can be observed in Figure 3(a). The convergence of state errors to zero in finite time is evident from Figure 3(b), and thus, it can be concluded that the master and slave system get anti-synchronized in finite time.



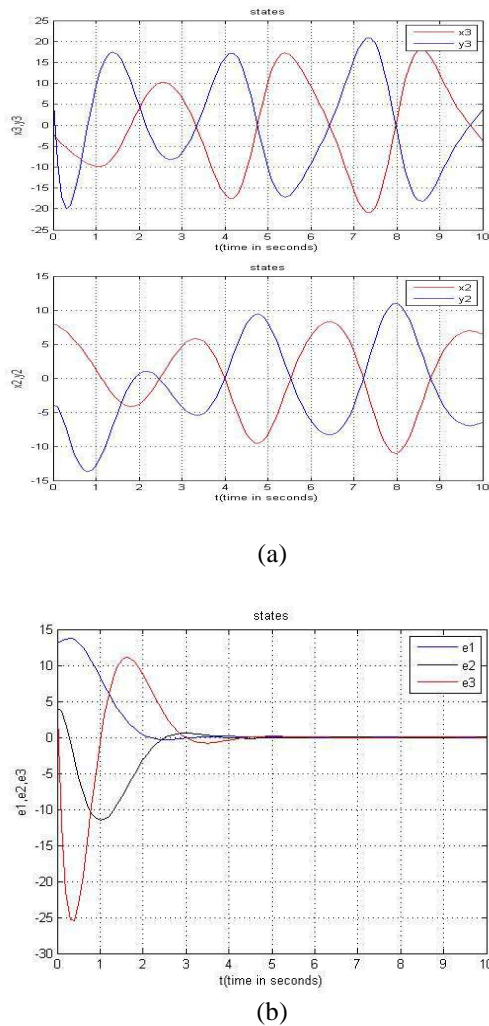


Fig.3. Anti-Synchronization of Arneodo system: (a) Anti-Synchronization of states and (b) Convergence of errors

5. CONCLUSION

In this paper, we performed new results for the synchronization and anti-synchronization of identical Arneodo chaotic systems (1980) using dynamic surface control (DSC) method. Numerical figures were shown by using MATLAB to illustrate the validity and successful operation of the DSC design for the synchronization and anti-synchronization of identical chaotic systems. DSC can be further utilized for the synchronization of two or more systems.

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